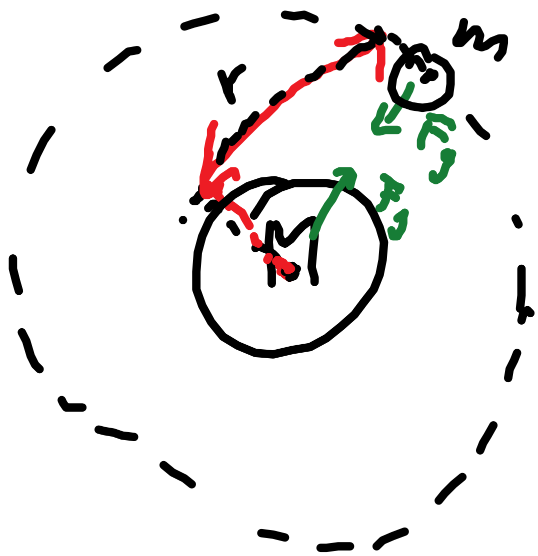
Monday, January 04, 2016
8:46 AM

Orbits and Energy
(universal energy is not in textbook so you will get a handout)

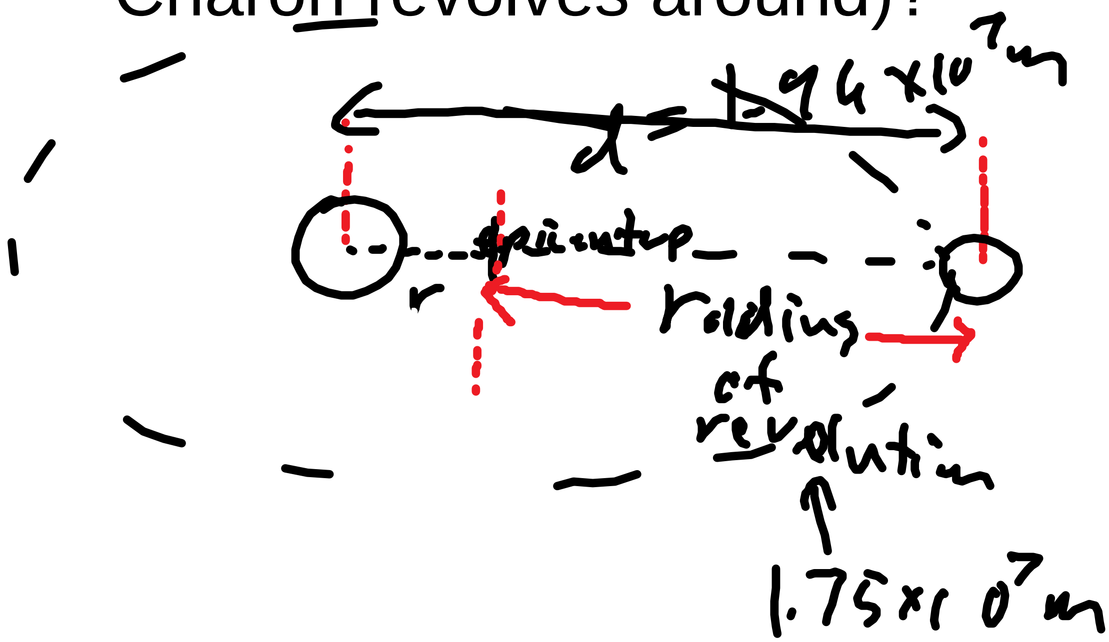
Look at a mass m in circular orbit around another mass, M.



Determine the equation for
a) force between the masses
b) the acceleration of mass, m in terms of M, m and r.
c) the velocity of mass m
d) the period of revolution of mass m
e) calculate a)-d) for Charon as it

revolves around Pluto.
Pluto mass 1.31×10^{22}kg
Charon mass 1.90×10^{21}kg
Charon's orbital radius 1.75×10^{7} m
distance to centre of Pluto is 1.96×10^{7}m

f) What is the period of Pluto around the epicentre (the same point Charon revolves around)?



https://en.wikipedia.org/wiki/Charon_(moon)#/media/File:Pluto-Charon_System.gif

$F_g=GMm/r^2$
$G=6.67 \times 10^{-11}Nm^2/kg^2$

$a_c = v^2/r = 4\pi^2 r/T^2$

if you are in an orbit

$F_g = F_c$

a) $F_g = GMm/r^2$

$= 6.67 \times 10^{-11} Nm^2/kg^2\ (1.31 \times 10^{22} kg)\ (1.9 \times 10^{21} kg)\ /(1.96 \times 10^{7} m)^2$

watch out, r isn't always radius, it is the distance between the centres

$= 4.3 \times 10^{18}$ N 1.1×10^{21}

b) $a = F/m = 2.274495 \times 10^{-3}$

$= 2.3 \times 10^{-3}\ m/s^2$

c) $v^2/r = a$ $v = \text{root}\ (ar) = \text{root}\ (2.3 \times 10^{-3} m/s^2\ 1.75 \times 10^{7} m\ )$

$v = 2.0 \times 10^{2}\ m/s$

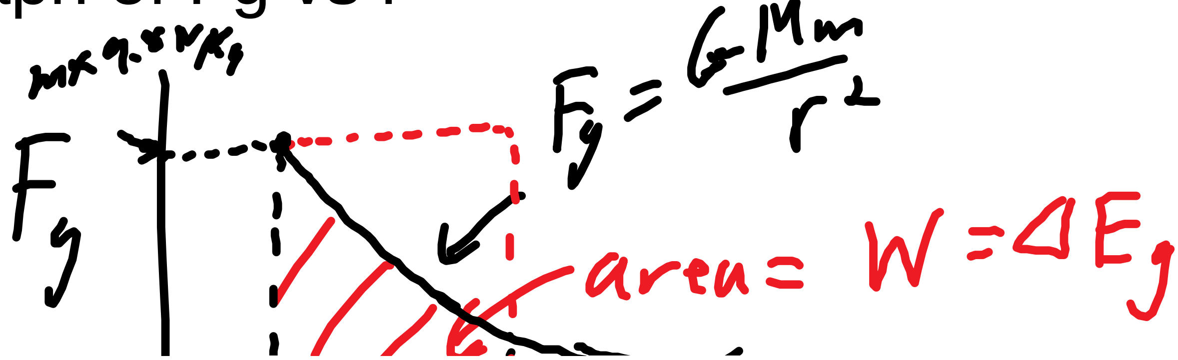
d) $a = 4\pi^2 r/T^2$

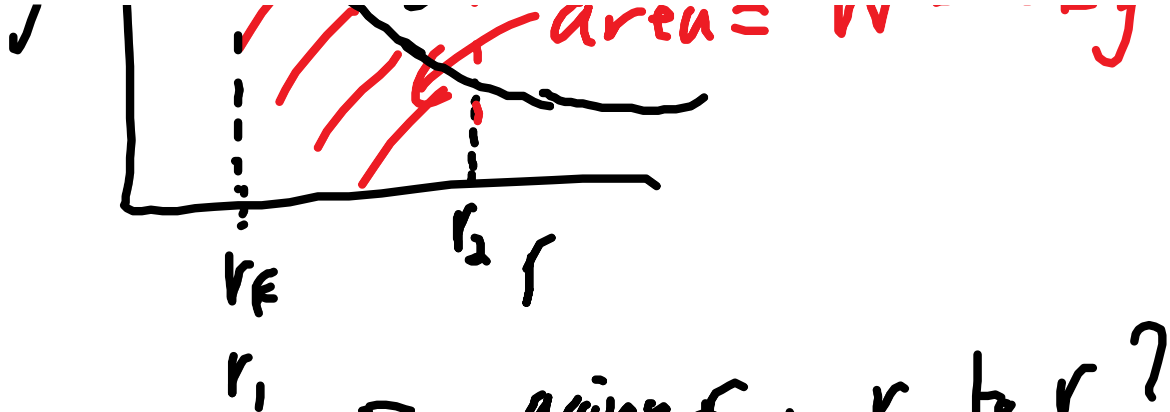
$T = \text{root}\ (4\pi^2 r/a) = 5.5 \times 10^{5} s = 6.4$ days

f) same as Charon

Gravitational Energy, Eg
If g is uniform (h is small relative to the planet radius)
E_g = mgh
where m is mass of the object
g is the gravitational field strength
(near Earth, g= 9.80N/kg)
h is the height relative to any arbitrary point.
if h is relative to the floor, Eg is positive and if h is relative to the ceiling, Eg is negative.
What if h is large relative to Earth, so g changes?

Graph of Fg vs r





What is the ΔE_g going from r_1 to r_2?

= Work done against gravity

$W = Fd$ if F is constant

= area under graph if F is not constant

$$= \int F_g \, dr = GMm \int_{r_1}^{r_2} r^{-2} \, dr$$

constants

$$\frac{dx^2}{dx} = 2x \qquad \int x \, dx = \frac{1}{2} x^2$$

$$W_g = \Delta E_g = -GMm\left(\frac{1}{r_2} - \frac{1}{r}\right)$$

where is $\frac{GMm}{r} = 0$?

as $r \rightarrow \infty$ infinity $\quad E_g \rightarrow 0$

So, we will define gravitational energy, Eg as the work done against gravity moving a mass m from infinity to r.

$\boxed{E_g = - GMm/r}$ relative to 0 at infinity
the negative sign

a) How much gravitational energy does a 1000 kg rocket have on the surface of the Earth?
b) How much energy is required to send the rocket up 400 km?
c) if you calculated b using mgh, what is the difference in energy? Wrong

a) Eg = -GMm/r $\quad$ Earth's mass
$-6.67 \times 10^{-11} \text{Nm}^{\cancel{2}}/\cancel{\text{kg}^2} (5.98 \times 10^{24} \cancel{\text{kg}})$
$(1000\text{kg})/6.38 \times 10^6\text{m}$ $\quad$ Earth

$-6.67 \times 10^{-11}N\cancel{m^2}/\cancel{kg^2}\ (5.98 \times 10^{24}\cancel{kg})$
$(1000\cancel{kg})/6.38 \times 10^{6}\cancel{m}$ ← radius of Earth
$= -6.25 \times 10^{10}$ Nm
$= -6.25 \times 10^{10}$ J

b) $Eg_f - Eg_i =$

$Eg_f =$
$-6.67 \times 10^{-11}Nm^2/kg^2\ (5.98 \times 10^{24}kg)$
$(1000kg)/(\underbrace{6.38 \times 10^{6}m}_{r_E} + \underbrace{400\,000m}_{h})$
-5.88×10^{10}J
$Eg_f - Eg_i = -5.88 \times 10^{10}J - (-6.25 \times 10^{10}\ J)$\
$= 3.7 \times 10^{9}$ J

c) mgh = 1000 kg 9.8N/kg x 400 000m
$= 3.9 \times 10^{9}$ J
off by 2×10^{8}J